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The Physics of Z' bosons *

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ABSTRACT

In these lectures we review the simplest gauge extensions of the standard model, and the present and future limits on new weak interactions.

1. Introduction

The physics of Z' bosons addresses two main questions: Which is the reason for extra weak interactions ? How does a new neutral gauge boson manifest at the electroweak scale ? The first question is a model building one and it is believed to be related with the physics at the Planck scale. The second question is a phenomenological one and it will be our main interest in these lectures.

The standard model gives a description of Nature below $100\ GeV$ in impressive agreement with experiment [1]. As a result present data put non-trivial bounds on possible extensions of the standard model, in particular on any electroweak gauge extension. However, these limits are very model dependent and rather weak for some models. There is a large literature on the subject to which we refer in the text. For more detailed lectures on Z' bounds from LEP see T. Riemann at this School [2].

We organize these lectures revising first the simplest (E_6) gauge extensions of the standard model. In a short version of the second lecture we summarize present indirect limits on new gauge interactions. Whereas the third lecture is devoted to recent developments on the diagnostics of Z' gauge couplings to known quarks and leptons at large hadron and lepton colliders.

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2. Simplest (E_6) gauge extensions of the standard model

The purpose of this lecture is to introduce a general, convenient parametrization of (minimal) extended electroweak models in order to compare with experiment.

The simplest gauge extension of the standard model results from adding only one extra neutral gauge boson, Z'. The corresponding gauge group enlarges the electroweak standard model by including one extra $U'(1)$ factor: $SU(2)_L \times U(1)_Y \times U'(1)$. In this case and in what follows we will assume that the new charge Q' commutes with the $SU(2)_L$ generators T_i: $[Q', T_i] = 0$. We will also assume family universality.

Prior to the introduction of popular (E_6) gauge extensions of the standard model, let us discuss the general parametrization of the gauge interactions for a gauge group with two abelian $U(1)$ factors [3].

2.1 General tree level lagrangian for $U^1(1) \times U^2(1)$

For the sake of definiteness we consider n massless Weyl fermions $\psi_k, k = 1, ..., n$, with different $U^a(1), a = 1, 2,$ charges Y^a_k, and then unmixed. Calling $A^b_\mu, b = 1, 2,$ the massless gauge bosons, the most general lagrangian reads

$$\mathcal{L} = -\frac{1}{4} F^b_{\mu\nu} F^{b\mu\nu} + \bar{\psi}_k i \not{\partial} \psi_k + \bar{\psi}_k \gamma^\mu Y^a_k \psi_k g_{ab} A^b_\mu \tag{1}$$

where summation on repeated indices is understood. This would be the standard QED lagrangian but for two different gauge interactions if g_{ab} were diagonal. However, g_{ab} is not diagonal in general. There is no symmetry reason for being so, and, moreover, the off-diagonal gauge couplings are observable !

- Looking for known examples, the neutral sector of the standard model $U(1)_{T_3} \times U(1)_Y$ is *not general enough*. In this case g_{ab} is diagonal ! But there is a symmetry reason: *gauge invariance*. $U(1)_{T_3}$ is within $SU(2)_L$ and no mixing among the non-abelian $SU(2)_L$ and the abelian $U(1)_Y$ massless gauge bosons is allowed.

- Usually when illustrating the electroweak gauge extensions of the standard model by adding an extra $U'(1)$, the possible mixing with the standard $U(1)_Y$ factor is *not discussed*. This is only *justified under certain assumptions*. At any rate, in the particular cases usually discussed the numerical effects of this mixing are *small*.

- These off-diagonal gauge couplings are necessary when working to higher orders in perturbation theory for quantum corrections *bring them back*. They are also necessary to renormalize the theory because *a full set of renormalization conditions* is required.

- This mixing allows for a general parametrization of an extra neutral gauge interaction and for the possibility of *fitting* to precise data *minimazing within*

the corresponding class of models. This class includes the popular E_6 models used often for illustration.

Using the invariance of the previous lagrangian under the rotation of the gauge bosons we can always assume that the matrix g_{ab} is triangular, $g_{ab} = 0, a > b$. The resulting $\frac{1}{2}N(N+1)$ ($= 3$ for $N = 2$) couplings become then physical. Once a basis for the charges Y_k^a defining $U^1(1) \times U^2(1)$ is chosen, the fixed triangular matrix g_{ab} can be measured, and it is not correct in general to assume it to be diagonal.

2.2 General tree level parametrization for $SU(2)_L \times U(1)_Y \times U'(1)$

This parametrization was first introduced in Ref. [4]. The explicit form discussed below was presented in detail in Ref. [5]. The interacting lagrangian in Eq. (1) for $SU(2)_L \times U(1)_Y \times U'(1)$ and the standard model fermions reads:

$$-\mathcal{L}_{NC} = \bar{\psi}_k \gamma^\mu \{T_{3k} g W_{3\mu} + Y_k g'_{11} B'_\mu + Y_k g'_{12} Z'_{2\mu} + Q'_k g'_{21} B'_\mu + Q'_k g'_{22} Z'_{2\mu}\} \psi_k \qquad (2)$$

where T_3 and Y are the standard isospin and hypercharge charges (see Table 1) and Q' is the extra $U'(1)$ charge. As emphasized above g'_{ab} can be made triangular by rotating the abelian gauge bosons:

$$-\mathcal{L}_{NC} = \bar{\psi}_k \gamma^\mu \{T_{3k} g W_{3\mu} + Y_k g_{11} B_\mu + Y_k g_{12} Z_{2\mu} + Q'_k g_{22} Z_{2\mu}\} \psi_k \qquad (3)$$

where $B'_\mu = c' B_\mu - s' Z_{2\mu}, Z'_{2\mu} = s' B_\mu + c' Z_{2\mu}$. c' and s' are the cosinus and sinus of the corresponding rotation angle fulfilling $g_{21} = g'_{21} c' + g'_{22} s' = 0$; whereas $g_{11} = g'_{11} c' + g'_{12} s', g_{12} = -g'_{11} s' + g'_{12} c', g_{22} = -g'_{21} s' + g'_{22} c'$ are *measurable* couplings. The photon is defined as a combination of $W_{3\mu}$ and B_μ: $W_{3\mu} = s_W A_\mu + c_W Z_{1\mu}, B_\mu = c_W A_\mu - s_W Z_{1\mu}$. The extended neutral current lagrangian for the standard Dirac fermions can then be written in general [6]:

$$-\mathcal{L}_{NC} = e \sum_i q_i \bar{\psi}_i \gamma^\mu \psi_i A_\mu + \frac{g}{2c_W} \sum_i \bar{\psi}_i \gamma^\mu (v^i - a^i \gamma_5) \psi_i Z_{1\mu} + \frac{g}{2c_W} \sum_i \bar{\psi}_i \gamma^\mu (v'^i - a'^i \gamma_5) \psi_i Z_{2\mu} \quad (4)$$

with the charges given in Table 2, $e = \frac{\sqrt{\frac{3}{5}} g g_{11}}{\sqrt{g^2 + \frac{3}{5} g_{11}^2}}$ is the positron charge, and $s_W \equiv sin\theta_W = \frac{e}{g}$ is the electroweak mixing angle ($c_W = cos\theta_W$). Hence any extra neutral gauge interaction can be written in the current eigenstate gauge boson basis adding an extra term for the new gauge boson similar to the standard model one but allowing for a linear combination of the extra charge Q' and the standard model hypercharge Y: $g_{22} Q' + g_{12} Y$ in Eq. (3). Once the charge of the extra $U'(1)$, Q', is specified, v'^i and a'^i in Eq. (4) can be calculated (see below for popular models). They are functions of $g_2 \equiv g_{22}$ and $\frac{g_{12}}{g_2}$. However, g_2 is often the only coupling strength introduced in the literature for the new interaction. It is also often taken equal or similar to $g_{11} = \sqrt{\frac{5}{3}} \frac{e}{c_W}$. But in general the neutral current lagrangian for $SU(2)_L \times U(1)_Y \times U'(1)$ depends on 4 independent, measurable parameters: e, s_W, g_2 and $\frac{g_{12}}{g_2}$.

	T_3	$\sqrt{\frac{5}{3}}Y$	Q'
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\left.\begin{matrix}\frac{1}{2} \\ -\frac{1}{2}\end{matrix}\right\}$	$\frac{1}{6}$	q'_{q_L}
u_L^c	0	$-\frac{2}{3}$	$q'_{u_L^c}$
d_L^c	0	$\frac{1}{3}$	$q'_{d_L^c}$
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$\left.\begin{matrix}\frac{1}{2} \\ -\frac{1}{2}\end{matrix}\right\}$	$-\frac{1}{2}$	q'_{l_L}
e_L^c	0	1	$q'_{e_L^c}$

Table 1. Isospin T_3, hypercharge Y, and new charge Q' assignments for ordinary fermions.

	Q	V	A	$\frac{g}{c_W}V'$	$\frac{g}{c_W}A'$
u	$\frac{2}{3}$	$\frac{1}{2}-\frac{4}{3}sin^2\theta_W$	$\frac{1}{2}$	$g_2[(q'_{q_L}-q'_{u_L^c})+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}\frac{5}{6}]$	$g_2[(q'_{q_L}+q'_{u_L^c})+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-\frac{1}{2})]$
d	$-\frac{1}{3}$	$-\frac{1}{2}+\frac{2}{3}sin^2\theta_W$	$-\frac{1}{2}$	$g_2[(q'_{q_L}-q'_{d_L^c})+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-\frac{1}{6})]$	$g_2[(q'_{q_L}+q'_{d_L^c})+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}\frac{1}{2}]$
ν	0	$\frac{1}{2}$	$\frac{1}{2}$	$g_2[q'_{l_L}+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-\frac{1}{2})]$	$g_2[q'_{l_L}+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-\frac{1}{2})]$
e	-1	$-\frac{1}{2}+2sin^2\theta_W$	$-\frac{1}{2}$	$g_2[(q'_{l_L}-q'_{e_L^c})+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-\frac{3}{2})]$	$g_2[(q'_{l_L}+q'_{e_L^c})+\frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}\frac{1}{2}]$

Table 2. Electric charge, $Q = T_3 + \sqrt{\frac{5}{3}}Y$, and vector and axial couplings for $Z_{1\mu}$: $v^i = t_{3i} - 2q_i sin^2\theta_W, a^i = t_{3i}$, and $Z_{2\mu}$: $\frac{g}{c_W}v'^i = g_2[(q'_{iL}+q'_{iR}) + \frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-t_{3i} + 2q_i)], \frac{g}{c_W}a'^i = g_2[(q'_{iL}-q'_{iR}) + \frac{g_{12}}{g_2}\sqrt{\frac{3}{5}}(-t_{3i})]$, for ordinary fermions in Table 1.

2.3 $SO(10), E_6$, superstring-motivated and LR models

Popular extended electroweak models are particular cases of Eq. (4) and Table 2:

- model χ occurs when $SO(10)$ breaks down to $SU(5)\times U(1)_\chi$;
- model ψ occurs when E_6 breaks down to $SO(10)\times U(1)_\psi$; and
- model η occurs in superstring-inspired models in which E_6 breaks directly to a rank 5 group [7].

The corresponding charges are given in Table 3.

Q'	$2\sqrt{10}Q_\chi$	$2\sqrt{6}Q_\psi$	$2\sqrt{15}Q_\eta$
q'_{q_L}	-1	1	-2
$q'_{u_L^c}$	-1	1	-2
$q'_{d_L^c}$	3	1	1
q'_{l_L}	3	1	1
$q'_{e_L^c}$	-1	1	-2

Table 3. Popular Z' model charges.

- The general E_6 extra $U(1)$ depends on a mixing angle, β, which fixes the combination of two extra, independent $U(1)$'s arbitrarily chosen within E_6, *e.g.*, $U(1)_\chi$ and $U(1)_\psi$: $Q_{E_6} = cos\beta Q_\chi + sin\beta Q_\psi$.

χ, ψ and η correspond to $\beta = 0, \frac{\pi}{2}, -arctan\sqrt{\frac{5}{3}}$, respectively. In all these models $g_2 = \sqrt{\frac{5}{3}}\frac{e}{c_W}, g_{12} = 0$. The neutral current lagrangian for left-right (LR) models is also a particular case of Eq. (4) and Table 2.

- LR models are parametrized by the ratio $\kappa = \frac{g_R}{g_L}$ of the gauge couplings $g_{L,R}$ for $SU(2)_{L,R}$, respectively. $\kappa > \frac{s_W}{c_W}$ for consistency [8].

The extra charge $Q_{LR} = Q_\chi$, because it is the only extra one within $SO(10)$. In this case, however, $g_2 = \frac{e}{c_W}\sqrt{\frac{2}{5}}\frac{\beta^2+1}{\beta}, \frac{g_{12}}{g_2} = \sqrt{\frac{1}{6}}\frac{3\beta^2-2}{\beta^2+1}, \beta = \sqrt{\kappa^2 cot^2\theta_W - 1}$. Although g_2 and $\frac{g_{12}}{g_2}$ are more convenient for comparison with experiment, unification of gauge couplings at a very high scale motivates an alternative parametrization [5]:

$$g_2 = \sqrt{\frac{5}{3}}\frac{e}{c_W}(\frac{c_1^2}{\lambda} + \lambda s_1^2), \frac{g_{12}}{g_2} = \frac{s_1 c_1(\lambda^2 - 1)}{s_1^2\lambda^2 + c_1^2}, \tag{5}$$

where $s_1(c_1) \equiv sin\theta_1(cos\theta_1), \theta_1 \in [-\frac{\pi}{2}, \frac{\pi}{2}), \lambda \geq 1$. β is also replaced by $\theta_2 = -\beta - arctan\sqrt{\frac{5}{3}}$. (In any case care must be taken with the different ranges of variation and sign conventions existing in the literature.)

2.4 Z' lagrangian parametrization

In extended electroweak models heavy gauge bosons can mix. Thus,

$$Z_{1\mu} = c_3 Z_\mu + s_3 Z'_\mu, Z_{2\mu} = -s_3 Z_\mu + c_3 Z'_\mu \tag{6}$$

in Eq. (4). Z is the observed heavy neutral gauge boson and Z' is the new gauge boson; whereas $s_3(c_3) \equiv sin\theta_3(cos\theta_3)$ is the $Z_1 Z'$ mixing.

2.5 Relevant couplings for large hadron and lepton colliders

We can look for indirect Z' effects in precise electroweak data and for direct Z' production at large colliders. Precise low energy and in particular LEP data test mainly the Z_μ current (see Eqs. (4, 6)):

$$\frac{g}{c_W}J^\mu = \frac{g}{2c_W}\sum_i \bar{\psi}_i\gamma^\mu[(v^i c_3 - v'^i s_3) - (a^i c_3 - a'^i s_3)\gamma_5]\psi_i. \tag{7}$$

As no indication for new effects beyond the standard model is found, it is deduced that only a small gauge boson mixing is allowed. (See Ref. [9] for a recent discussion. See also T. Riemann lectures at this School [2].) Present limits require $s_3 < 1\%$.

For Z' physics at future colliders more convenient sets of effective charges can be defined [10, 11]. Production bounds test mainly the Z'_μ current:

$$\frac{g}{c_W}J'^\mu = \frac{g}{2c_W}\sum_i \bar{\psi}_i\gamma^\mu[(v'^i c_3 + v^i s_3) - (a'^i c_3 + a^i s_3)\gamma_5]\psi_i. \tag{8}$$

The precision available at large colliders and the experimental limit on s_3 permit to neglect the $Z_1 Z'$ mixing. The Z' couplings to ordinary fermions is fixed by five normalized charges $\hat{g}^u_{L2} = \hat{g}^d_{L2} \equiv \hat{g}^q_{L2}, \hat{g}^u_{R2}, \hat{g}^d_{R2}, \hat{g}^\nu_{L2} = \hat{g}^e_{L2} \equiv \hat{g}^l_{L2}, \hat{g}^e_{R2}$ and the gauge coupling strength g_2:

$$g_2 \hat{g}^i_{L,R2} \equiv \frac{g}{2c_W}(v'^i \pm a'^i). \tag{9}$$

The signs of the charges will be hard to determine at hadron colliders. The following set of four normalized couplings is probed directly [10]:

$$\gamma^l_L \equiv \frac{(\hat{g}^l_{L2})^2}{(\hat{g}^l_{L2})^2 + (\hat{g}^e_{R2})^2}, \gamma^q_L \equiv \frac{(\hat{g}^q_{L2})^2}{(\hat{g}^l_{L2})^2 + (\hat{g}^e_{R2})^2}, \tilde{U} \equiv \frac{(\hat{g}^u_{R2})^2}{(\hat{g}^q_{L2})^2}, \tilde{D} \equiv \frac{(\hat{g}^d_{R2})^2}{(\hat{g}^q_{L2})^2}. \tag{10}$$

At large lepton colliders the main effects of a new gauge boson far off-shell result from its interference with the photon and the Z boson. Since the photon couplings are only vector-like and the l couplings to Z are almost axial, it turns out that the probes in the two-fermion final state channels single out the Z' leptonic couplings primarily in the combinations $\hat{g}^l_{L2} \pm \hat{g}^e_{R2}$. To trace the combinations of the normalized charges to which the probes are sensitive, it is advantageous to choose either of the two combinations to normalize the charges. We choose the $\hat{g}^l_{L2} - \hat{g}^e_{R2}$ combination, which turns out to be a convenient choice for popular models. We then define the following four independent normalized charges [11]:

$$P^e_V = \frac{\hat{g}^l_{L2} + \hat{g}^e_{R2}}{\hat{g}^l_{L2} - \hat{g}^e_{R2}},\ P^q_L = \frac{\hat{g}^q_{L2}}{\hat{g}^l_{L2} - \hat{g}^e_{R2}},\ P^{u,d}_R = \frac{\hat{g}^{u,d}_{R2}}{\hat{g}^q_{L2}}. \tag{11}$$

In Table 4 we give the values of the couplings in Eqs. (10, 11) for models χ, ψ, η and LR for the special value $\kappa = 1$.

	χ	ψ	η	LR
γ^l_L	0.9	0.5	0.2	0.36
γ^q_L	0.1	0.5	0.8	0.04
$\tilde{U}$	1	1	1	37
$\tilde{D}$	9	1	0.25	65
P^e_V	2	0	-3	-0.148
P^q_L	-0.5	0.5	2	-0.142
P^u_R	-1	-1	-1	-6.04
P^d_R	3	-1	0.5	8.04

Table 4. Couplings in Eqs. (10, 11) for models χ, ψ, η and LR (with $\kappa = 1$ and $s^2_W = 0.23$).

2.6 Model independent analyses and model building constraints

In this lecture we have revised the parametrization of an extra $U(1)$ interaction, specifying the values of the new couplings for typical models; and we have introduced convenient sets of normalized charges for comparison with experiment at hadron and lepton colliders. To finish we discuss some predictions for these couplings from unification near the Planck scale.

Models with an extra abelian gauge interaction are fixed by two extra gauge couplings: g_2, g_{12}. Their values near M_Z result from their evolution from the Planck down to the electroweak scale. Hence they are a window to all intermediate scales and matter contents in between. For illustration we gathered in Table 5 the λ, θ_1 values at the electroweak scale for some (grand) unified models [12], as well as the corresponding g_2, g_{12} values. In all cases $g_{12} = 0$ at the unification scale. We also quote the values for the χ and LR models. The (grand) unified models assume a minimal matter content and $\alpha_C = 0.12, \alpha_E = 128^{-1}$ and $s_W^2 = 0.23$ (which fixes the intermediate scale) at M_Z. The required intermediate and grand unified scales for these examples are probably too low/large, but there are many non-minimal matter contents which give acceptable models and a large variety of λ, θ_1 (g_2, g_{12}) values [13]. The χ, LR models with a minimal matter content and no intermediate scale do not unify. In conclusion, if a new Z' exists and then λ, θ_1 can be measured, they will provide precious information on the physics at and well beyond the TeV scale.

$G \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$	λ	θ_1	$\frac{g_2}{\sqrt{\frac{5}{3}}\frac{e}{c_W}}$	$\frac{g_{12}}{g_2}$
$B3 : SU(4)_C \times SU(2)_L \times SU(2) \times U(1)$	1.37	1.20	1.28	0.17
$B4 : SU(3)_C \times SU(3)_L \times SU(2) \times U(1)$	1.18	-1.02	1.09	-0.14
$B7 : SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)$	1.08	0.17	0.93	0.03
$B8 : SU(3)_C \times SU(3)_L \times SU(2)_R \times U(1)$	1.23	-0.69	0.98	-0.21
$C1, ..., C5, D : SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$	1.04	-0.89	1.01	-0.04
χ	1	$-$	1	0
LR	1.88	0.68	1.07	0.62

Table 5. λ, θ_1 values and the corresponding gauge couplings g_2, g_{12} for some unified models and for the χ, LR models.

3. Indirect limits on extra gauge interactions

We give a short summary of the second lecture. At present there are only limits on new weak interactions. These are direct bounds on Z' production at TEVATRON (see next lecture) and indirect bounds from fits to precise electroweak data. (We do not consider cosmological bounds in these lectures.) Indirect limits are very model dependent. The experimental precision is at the level of radiative corrections and new effects must be very small. Hence, one must wonder not only about modifications of tree level expressions but about extra loop contributions. Enlarging the standard model consistently often requires to add many new particles and then many new parameters. As a matter of fact often indirect Z' limits must be understood as an order of magnitude estimate.

In practice the extra interaction is taken into account in fits at tree level only. This approximation is better when the limits are more stringent [14]. There are two approaches for reducing the number of free parameters: stick with simple, well-defined models and/or use phenomenologically relevant parameters. An example of the first approach can be found in Ref. [15]. The simplest class of extended models within E_6 is the class defined by the extra charge within $SO(10) : Q_\chi = Q_{LR}$ in Table

3. These models contain right-handed neutrinos, which we assume to be heavy, and a minimal Higgs sector which makes the Z_1Z' mixing a well-defined function of the Z' mass. The class is then fixed by the Z' mass, $M_{Z'}$, and two gauge couplings, g_2, g_{12} parametrized by λ, θ_1 (see Eq. (5)). See Table 5 for examples. In Ref. [15] the $M_{Z'}$ lower bounds ($90\% c.l.$ for one variable) implied by 1991 precise electroweak data are plotted as a function of λ and θ_1. The standard model limit corresponds to $\theta_1 = 0, \lambda \to \infty$.

The second approach prefers a parametrization adapted to available data. Low energy electroweak data as well as LEP data are mainly sensitive to the Z' mass and the Z_1Z' mixing. These redefine the ρ parameter and the Z current:

$$\begin{aligned} &\rho = (1 - \frac{s_3^2(M_{Z'}^2 - M_Z^2)c_W^2}{M_W^2})^{-1}, \\ &J_Z = c_3 J_{Z_1} - s_3 J_{Z_2}. \end{aligned} \tag{12}$$

ρ (J_Z) is mainly constrained by low energy (LEP) data. A detailed analysis is more involved. The present limits can be summarized saying that once fixed the Z' charges, for a large class of models present data require [9,16]

$$M_{Z'} > 200\ GeV,\ \ s_3 < 0.01. \tag{13}$$

See T. Riemann lectures for a detailed discussion of these bounds, in particular from LEP [2].

4. Determination of Z′ gauge couplings to ordinary fermions at future colliders

In this lecture we study the potential of large hadron and lepton colliders for the discovery of a new neutral gauge boson and for a model independent determination of its couplings. We use popular models for illustration only.

4.1 Hadron colliders

If heavy gauge bosons turn out to have a mass in the few TeV region, future hadron colliders, *e.g.* LHC, would be an ideal place to discover and study them. In the main production channels, $p\bar{p}^{)} \to Z' \to l\bar{l}(l = e, \mu)$, one would be able to measure

- the mass $M_{Z'}$,
- the width $\Gamma_{Z'}$ and
- the total cross section σ_{ll}, as well as
- the forward-backward asymmetry A_{FB} [5,17] and
- the ratio of cross-sections in different rapidity bins r_{y_1} [10,18].

The Z' can also decay into WW and ZH. These decays are observed, however, as four fermion final states for W, Z, H decay into two fermions. (A heavy Higgs decays mainly into two gauge bosons.) Hence, the next dominant Z' decays are those

involving four fermions. They have small cross sections and are difficult to isolate in general, but they are important to determine the Z' couplings [10]. The main probes are

- rare decays $W \to W l \nu_l$ [19,20] and
- associated productions $pp \to Z'V$ with $V = Z, W, \gamma$ [21].

4.1.1 Two fermion final states: $p^{(}\bar{p}^{)} \to \gamma, Z, Z' \to l\bar{l}X; jetjetX$

The unpolarized differential cross section for the process $p^{(}\bar{p}^{)} \to \gamma, Z, Z' \to l\bar{l}X$ (with l a definite charged lepton) depends on the lepton-antilepton invariant mass M, where $M^2 = (p_l + p_{\bar{l}})^2$, on the rapidity y, where $\frac{M}{2}(e^y - e^{-y}) = (p_l + p_{\bar{l}})_{longitudinal}$, and on θ^*, which is the scattering angle pl in the center of mass of the parton system $q\bar{q}$ [5,17]. The general form of this cross section is, for tree level amplitudes, (quark and lepton masses are neglected)

$$\frac{d\sigma}{dMdydcos\theta^*} = \sum_{q=u,c,t,d,s,b} \frac{M}{192\pi}[g_q^S(y,M)S_q(M)(1+cos^2\theta^*) + g_q^A(y,M)A_q(M)2cos\theta^*], \tag{14}$$

where S_q, A_q (which are the only model dependent quantities) involve the gauge couplings to quarks and leptons,

$$\begin{aligned} S_q &= \sum_{\alpha,\beta=\gamma,Z,Z'} \frac{(g^q_{L\alpha}g^q_{L\beta} + g^q_{R\alpha}g^q_{R\beta})(g^l_{L\alpha}g^l_{L\beta} + g^l_{R\alpha}g^l_{R\beta})}{(M^2 - M^2_\alpha + iM_\alpha\Gamma_\alpha)(M^2 - M^2_\beta - iM_\beta\Gamma_\beta)}, \\ A_q &= \sum_{\alpha,\beta=\gamma,Z,Z'} \frac{(g^q_{L\alpha}g^q_{L\beta} - g^q_{R\alpha}g^q_{R\beta})(g^l_{L\alpha}g^l_{L\beta} - g^l_{R\alpha}g^l_{R\beta})}{(M^2 - M^2_\alpha + iM_\alpha\Gamma_\alpha)(M^2 - M^2_\beta - iM_\beta\Gamma_\beta)}, \end{aligned} \tag{15}$$

and g_q^S, g_q^A are the parton distribution functions of the colliding hadrons [22],

$$g_q^{S,A}(y,M) = x_a x_b[f_q^{(a)}(x_a,M^2)f_{\bar{q}}^{(b)}(x_b,M^2) \pm f_{\bar{q}}^{(a)}(x_a,M^2)f_q^{(b)}(x_b,M^2)], \tag{16}$$

where the sign $\pm$ correspond to S, A. a and b are the two colliding hadrons at the center of mass energy $\sqrt{s}$; x_a and x_b being the momentum fractions of the colliding partons in a and b respectively, $x_{a,b} = \frac{M}{\sqrt{s}}e^{\pm y}$. The gauge couplings were discussed in the first lecture:

$$\begin{aligned} &g^i_{L,R\ \gamma} \equiv eq_i, \\ &g^i_{L,R\ Z} \equiv \frac{g}{2c_W}[(v^i c_3 - v'^i s_3) \pm (a^i c_3 - a'^i s_3)], \\ &g^i_{L,R\ Z'} \equiv g_2 \hat{g}^i_{L,R\ Z'} = \frac{g}{2c_W}[(v'^i c_3 + v^i s_3) \pm (a'^i c_3 + a^i s_3)]. \end{aligned} \tag{17}$$

At large hadron colliders the γ, Z contributions and the $Z_1 Z'$ mixing can be neglected in two fermion channels. The statistics eventually available and the limit on s_3 from precise electroweak data discussed in the previous lecture allow for considering only the Z' contribution (no γ, Z interference) with zero gauge boson mixing ($s_3 = 0$). Then Eqs. (9, 10) apply.

In the absence of a positive signal the Z' total cross section

$$\sigma_{ll} = \int_0^{\sqrt{s}} dM \int_{ln\frac{M}{\sqrt{s}}}^{ln\frac{\sqrt{s}}{M}} dy \int_{-1}^{1} dcos\theta^* \frac{d\sigma}{dMdydcos\theta^*}, \text{ (with } \alpha,\beta = Z' \text{ in Eq. (15),)} \tag{18}$$

for each given model (with the gauge coupling strength and $\Gamma_{Z'}$ also fixed) puts a limit on $M_{Z'}$. TEVATRON limits on an excess of isolated lepton pairs with large invariant mass translate into limits on $M_{Z'}$. In Table 6 we gather the Z' couplings to ordinary fermions and the minimal Z' width for the popular models discussed in the first lecture. For an integrated luminosity of $4\ pb^{-1}$ the $M_{Z'}$ (95% $c.l.$) bounds for these models are [6,23,24]

$$M_{\chi,\psi,\eta,LR} > 340, 320, 340, 310\ GeV. \tag{19}$$

	χ	ψ	η	LR
g_2	0.461	0.461	0.461	0.493
$\hat{g}^l_{L\ Z'} = \hat{g}^\nu_{L\ Z'} = \hat{g}^e_{L\ Z'}$	$\frac{3}{2\sqrt{10}}$	$\frac{1}{2\sqrt{6}}$	$\frac{1}{2\sqrt{15}}$	0.253
$\hat{g}^e_{R\ Z'}$	$\frac{1}{2\sqrt{10}}$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{\sqrt{15}}$	−0.341
$\hat{g}^q_{L\ Z'} = \hat{g}^u_{L\ Z'} = \hat{g}^d_{L\ Z'}$	$-\frac{1}{2\sqrt{10}}$	$\frac{1}{2\sqrt{6}}$	$-\frac{1}{\sqrt{15}}$	−0.084
$\hat{g}^u_{R\ Z'}$	$\frac{1}{2\sqrt{10}}$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{\sqrt{15}}$	0.509
$\hat{g}^d_{R\ Z'}$	$-\frac{3}{2\sqrt{10}}$	$-\frac{1}{2\sqrt{6}}$	$-\frac{1}{2\sqrt{15}}$	−0.678
$\frac{\Gamma_{Z'}}{M_{Z'}}$	0.012	0.006	0.007	0.021

Table 6. Popular Z' model couplings $g^i_{L,R\ Z'} = g_2\hat{g}^i_{L,R\ Z'}$ and widths $\Gamma_{Z'}$.

If a new gauge boson with a mass $\leq 5\ TeV$ exits and couples to ordinary fermions with a sizeable strength ($g_2 \sim 0.1$), it should be observed in the two lepton channel at LHC. Once a new Z' is observed in the two lepton channel and its mass $M_{Z'}$ and width $\Gamma_{Z'}$ measured, one must try to measure the Z' couplings to fermions in a model independent way. With no γ, Z interference no determination of the signs of the Z' charges is possible (see Eqs. (14,15)). Therefore it is convenient to use the gauge coupling strength g_2 and four normalized couplings with no sign ambiguity, $e.g.$ $\gamma^l_L, \gamma^q_L, \tilde{U}, \tilde{D}$ in Eq. (10) (their values for typical models were given in Table 4), as free parameters. What matters is which couplings can be measured at LHC. In the two lepton channel the relevant observables are the total cross section σ_{ll}, the forward-backward asymmetry

$$\begin{aligned} A_{FB} &= \frac{\int dM(\int_0^{ln\frac{\sqrt{s}}{M}} - \int_{ln\frac{M}{\sqrt{s}}}^0)dy(\int_0^1 - \int_{-1}^0)dcos\theta^* \frac{d\sigma}{dMdydcos\theta^*}}{\sigma_{ll}} \\ &= 0.38(2\gamma^l_L - 1)\frac{1 - 0.75\tilde{U} - 0.25\tilde{D}}{1 + 0.68\tilde{U} + 0.32\tilde{D}}, \end{aligned} \tag{20}$$

and the ratio of cross-sections in different rapidity bins (y_1 is chosen in a range $0 < y_1 < y_{max}$ so that the number of events in the two bins are comparable, $y_1 = 1$ at LHC where $\sqrt{s} = 16\ TeV$) [10,18]

$$r_{y_1} = \frac{\int dM \int_{-y_1}^{y_1} dy \int dcos\theta^* \frac{d\sigma}{dMdydcos\theta^*}}{\int dM(\int_{-y_{max}}^{-y_1} + \int_{y_1}^{y_{max}})dy \int dcos\theta^* \frac{d\sigma}{dMdydcos\theta^*}} = 1.55\frac{1+0.64\tilde{U}+0.36\tilde{D}}{1+0.73\tilde{U}+0.27\tilde{D}}. \quad (21)$$

Another observable is the ratio

$$A_{FB_{y_1}} = \frac{\int dM(\int_0^{y_1} - \int_{-y_1}^{0})dy(\int_0^1 - \int_{-1}^{0})dcos\theta^* \frac{d\sigma}{dMdydcos\theta^*}}{\int dM(\int_{y_1}^{y_{max}} - \int_{-y_{max}}^{-y_1})dy(\int_0^1 - \int_{-1}^{0})dcos\theta^* \frac{d\sigma}{dMdydcos\theta^*}} = 0.60\frac{1-0.73\tilde{U}-0.27\tilde{D}}{1-0.76\tilde{U}-0.24\tilde{D}}. \quad (22)$$

The numerical expressions above are calculated assuming $M_{Z'} = 1\ TeV$ and the EHLQ structure functions set 1 [22]. A numerical analysis shows that $A_{FB_{y_1}}$ is not a sensitive enough function of the gauge couplings to provide useful information for the projected luminosities [10]. Thus only three combinations out of five gauge couplings can be determined in the two lepton channel. In particular A_{FB}, r_{y_1}, and $A_{FB_{y_1}}$ fix two combinations out of the four normalized couplings. Other observables do not provide more information if the Z' interference with the γ, Z bosons is neglected [18]. At any rate, a poor energy resolution would erase any interference effects from data.

The two lepton final state does not constrain γ_L^q. This process is proportional to the product of the Z' coupling to quarks times the Z' coupling to leptons. Hence, a simultaneous increase (decrease) of the quark coupling and a decrease (increase) of the lepton coupling by the same factor does not change the Z' cross section into leptons at LHC. The relative size of the Z' couplings to quarks and leptons could be determined by a measurement of the Z' cross section into quark pairs. In particular, the ratio

$$\frac{1}{3}\frac{\sigma(pp\to Z'\to q\bar{q})}{\sigma(pp\to Z'\to l^+l^-)} = \gamma_L^q(2+\tilde{U}+\tilde{D}) \quad (23)$$

(counting all three families) would yield the left-handed quark coupling γ_L^q [10]. However, this appears difficult [25].

The τ polarization in $pp\to Z'\to\tau^+\tau^-$ would be another useful probe if it can be measured [26].

Similarly, if proton polarization were available the measurements of the corresponding asymmetries in $pp\to Z'\to l^+l^-$ would also be useful [27].

4.1.2 Four fermion final states: $Z'\to WW, ZH, f\bar{f}V; Z'V$

A new Z' with a mass $\leq 2\ TeV$ can also show up in four fermion decays [19,21,28]. The WW, ZH decays have a rate similar to the Z' decay into two fermions plus one heavy gauge boson, $f\bar{f}V, V = W, Z$. All of them are observed as four fermion decays; and then they must be considered together because they can interfere. Associated production $Z'V, V = Z, W, \gamma$, cross sections are of a similar size, too.

The Z' four fermion cross sections are relatively small on one hand, and on the other it is difficult to isolate the corresponding samples. More detailed analyses

(simulations) are still needed to decide on the actual relevance of these processes. Below we discuss the main four fermion cross sections, assuming full efficiency in the identification of the final states, and perform a fit to know how well the Z' couplings could be ideally measured, including only statistical errors. After, we comment on the isolation of four fermion signals.

- $Z' \to WW$ [28]. The Z' decay rate into W pairs is proportional to the square of the trilinear $Z'WW$ coupling $\delta_{Z'WW}$:

$$\Gamma(Z' \to WW) = \frac{M_{Z'}}{12\pi}\frac{\delta^2_{Z'WW}}{16\eta_W^2}(1-4\eta_W)^{\frac{3}{2}}(1+20\eta_W+12\eta_W^2), \tag{24}$$

  with $\eta_W \equiv \frac{M_W^2}{M_{Z'}^2}$. This trilinear coupling is equal to the Z_1Z' mixing times the Z_1WW coupling, which is the product of the $SU(2)_L$ coupling times the W_3Z_1 mixing, [29]:

$$\delta_{Z'WW} = s_3\frac{e}{s_W}c_W. \tag{25}$$

  In typical models $s_3 \sim \frac{M_Z^2}{M_{Z'}^2}$ and $\Gamma(Z' \to WW)$ is not very large. $pp \to Z' \to WW$ is the only process probing directly the gauge symmetry breaking coupling $\delta_{Z'WW}$ and then s_3. As W's are observed through their decays, to isolate this signal requires refined cuts (see below).

- $Z' \to ZH$ [30,31]. This decay is somewhat model dependent. We will not consider it here.

- $Z' \to f\bar{f}V$ [19]. Recently it has been emphasized that this decay rate is enhanced due to the collinear singularity associated to soft vector boson emission:

$$\Gamma(Z' \to f\bar{f}Z) = \frac{M_{Z'}(g_{LZ}^{f\ 2}g_{LZ'}^{f\ 2}+g_{RZ}^{f\ 2}g_{RZ'}^{f\ 2})}{192\pi^2}[ln^2\eta_Z + 3ln\eta_Z + 5 - \frac{\pi^2}{3} + O(\eta_Z)], \tag{26}$$

  with $\eta_Z \equiv \frac{M_Z^2}{M_{Z'}^2} << 1$. $\Gamma(Z' \to f\bar{f}Z)$ has a similar expression but M_Z must be replaced by M_W and the Z couplings by $g_{LW}^f = \frac{e}{\sqrt{2}s_W}, g_{RW}^f = 0$. The photon emission decay $Z' \to f\bar{f}\gamma$ is a correction of the Z' decay into $f\bar{f}$ and it does not provide new information on the Z' coupling to fermions.

- *Associated Z' production:* $Z'V$ [21]. These cross sections are typically 1% the Z' production cross section. The different electric charge of up and down quarks makes $Z'\gamma$ production at LHC a useful probe to measure the Z' couplings to quarks.

The only rare decay which provides new information on Z' couplings to fermions and which seems possible to isolate is $Z' \to l\nu W$ (see, however, Ref. [32]). Normalizing to the Z' decay into lepton pairs ($l = e, \mu$) and assuming $M_{Z'} = 1\ TeV$ and the EHLQ structure functions set 1, the four fermion probes can be written (as a function of the normalized charges defined above) at LHC [10]:

$$
\begin{aligned}
r_{l\nu W} &\equiv \frac{\sigma(pp \to Z' \to (W^{\pm} \to hadrons) l^{\mp}\nu)}{\sigma(pp \to Z' \to l^+l^-)} \\
&= 0.067\gamma_L^l, \\
R_{Z'Z} &\equiv \frac{\sigma(pp \to (Z' \to l^+l^-)Z)}{\sigma(pp \to Z' \to l^+l^-)} \\
&= 10^{-3}\frac{7.94 + 0.96\tilde{U} + 0.11\tilde{D}}{1 + 0.68\tilde{U} + 0.32\tilde{D}}, \\
R_{Z'W} &\equiv \frac{\sigma(pp \to (Z' \to l^+l^-)W)}{\sigma(pp \to Z' \to l^+l^-)} \\
&= 10^{-3}\frac{25.7}{1 + 0.68\tilde{U} + 0.32\tilde{D}}, \\
R_{Z'\gamma} &\equiv \frac{\sigma(pp \to (Z' \to l^+l^-)\gamma)}{\sigma(pp \to Z' \to l^+l^-)} \\
&= 10^{-3} 5.62\frac{1 + 0.89\tilde{U} + 0.11\tilde{D}}{1 + 0.68\tilde{U} + 0.32\tilde{D}}
\end{aligned}
\tag{27}
$$

(with a transverse momentum cut $p_t^{\gamma} > 50\ GeV$). As can be readily seen these expressions do not contain (constrain) γ_L^q, because they are quadratic in the Z' couplings to quarks as well as to leptons. Hence when fitted with the two lepton Z' observables, they fix three out of four normalized couplings, leaving γ_L^q unconstrained. In Table 7 we write the values of these observables for typical models. Errors are statistical only. (We assume an integrated luminosity of $10^5 pb^{-1}$.) Performing a fit to these values we can determine $\gamma_L^l, \tilde{U}, \tilde{D}$. The results are given in Table 8.

	χ	ψ	η	LR
A_{FB}	-0.134 ± 0.007	0.0 ± 0.016	-0.025 ± 0.014	0.098 ± 0.006
r_{y_1}	1.79 ± 0.02	1.55 ± 0.04	1.49 ± 0.03	1.62 ± 0.014
$A_{FB_{y_1}}$	0.68 ± 0.08		0.68 ± 0.88	0.61 ± 0.08
$r_{l\nu W}$	0.060 ± 0.0014	0.034 ± 0.002	0.013 ± 0.001	0.024 ± 0.0008
$R_{Z'Z}$	0.0022 ± 0.0002	0.0045 ± 0.0008	0.0051 ± 0.0007	0.0011 ± 0.0001
$R_{Z'W}$	0.0056 ± 0.0004	0.013 ± 0.001	0.015 ± 0.001	0.00055 ± 0.00010
$R_{Z'\gamma}$	0.0035 ± 0.0003	0.0056 ± 0.0009	0.0061 ± 0.0008	0.0049 ± 0.0003

Table 7. Values of two and four fermion Z' observables for popular models. Errors are statistical only. Error bars for $r_{y_1}, r_{l\nu W}, R_{Z'V}$ are for e plus μ channels, while $A_{FB}, A_{FB_{y_1}}$ are for e or μ.

	χ	ψ	η	LR
γ_L^l	0.9 ± 0.018	0.5 ± 0.03	0.2 ± 0.015	0.36 ± 0.007
γ_L^q	0.1	0.5	0.8	0.04
$\tilde{U}$	1 ± 0.18	1 ± 0.27	1 ± 0.14	37 ± 8.3
$\tilde{D}$	9 ± 0.61	1 ± 0.41	0.25 ± 0.29	65 ± 14

Table 8. Values of $\gamma_L^l, \gamma_L^q, \tilde{U}, \tilde{D}$ for the χ, ψ, η, LR models. The error bars indicate how well the coupling could be measured at the LHC for $M_{Z'} = 1\ TeV$.

Four fermion Z' decays have large backgrounds. No detailed simulation of the different Z' rare decays has been published. To illustrate the reduction factors

we have to pay to isolate a relatively clean sample, let us discuss the Z' decay into an electron and a muon plus missing energy [20]. To this signal contribute $Z' \rightarrow (W \rightarrow e\nu)(W \rightarrow \mu\nu), (W \rightarrow e\nu)\mu\nu, (W \rightarrow \mu\nu)e\nu$. Wondersome backgrounds are W^+W^- continuum, $Z' \rightarrow \tau\bar{\tau}$, and (QCD) $t\bar{t}$ production. In Fig. 1 of Ref. [20] there are plotted the missing transverse momentum, $\not{p}_t$, and $e\mu$ angle in the transverse plane, $\theta_t^{e\mu}$, distributions. They are simulations at the parton level. The signal corresponds to the χ model with $M_{Z'} = 1\ TeV$. The top background was estimated assuming a top with a mass of $150\ GeV$ and that t decays into bW, and W into $l\nu$, isotropically. How large the top quark background might be depends on how good the top quark reconstruction is. Lepton isolation is enforced demanding the distance in the lego plot

$$\Delta R_{ql} = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2} > 0.75; \tag{28}$$

whereas misidentification of the top (bottom) quark decay is taken into account requiring $p_t^{b,\bar{b}} < 50\ GeV$ and a good balancing of the transverse momentum of the sample

$$\vec{p}_t^{\ out} \equiv (\vec{p}_e + \vec{p}_\mu + \not{\vec{p}})_t < 25\ GeV. \tag{29}$$

The $\not{p}_t$ and $\theta_t^{e\mu}$ distributions examplify what we want to emphasize: Z' rare decays have large backgrounds at LHC. In particular, the extremely large QCD cross sections require, in order to isolate the signals, identification of isolated leptonic tracks and stringent cuts on transverse energy. To obtain a relatively clean $Z' \rightarrow e\mu\not{p}$ sample a set of cuts, which reduces the signal by at least a factor of 3, is needed; *e.g.* the 450 events produced reduce to 120 after requiring $\not{p}_t > 200\ GeV, p_t^{e,\mu} > 50\ GeV, cos\theta_t^{e\mu} > -0.95$.

4.2 Lepton colliders

If a new Z' exists with a mass much larger than the available center of mass energy, its main effects at a large lepton collider result from its interference with γ, Z. (The (tree level) cross section for $e^+e^- \rightarrow \gamma, Z, Z' \rightarrow f\bar{f}$ can be found in Ref. [33].) This interference allows for measuring all Z' gauge couplings to ordinary fermions, including their relative signs. A convenient set of four normalized charges, $P_V^l, P_L^q, P_R^u, P_R^d$, was defined in Eq. (11). A fit to $e^+e^- \rightarrow f\bar{f}, f = l, u, d,$ observables is presented for $M_{Z'} = 1\ TeV$ at NLC (center of mass energy $\sqrt{s} = 500\ GeV$ and integrated luminosity $\mathcal{L}_{int} = 20 fb^{-1}$) in Ref. [11]. (See also Ref. [34].) The values used in the fit for the different observables correspond to typical models. Errors are only statistical. The results of the fit are gathered in Table 9. 100% efficiency for the heavy flavour tagging and 100% polarization for the electron beam are assumed. (Error bars in parentheses correspond to unpolarized electron beams.)

	χ	ψ	η	LR
P_V^e	$2 \pm 0.08(0.26)$	$0 \pm 0.04(1.5)$	$-3 \pm 0.5(1.1)$	$-0.15 \pm 0.018(0.072)$
P_L^q	$-0.5 \pm 0.04(0.10)$	$0.5 \pm 0.10(0.2)$	$2 \pm 0.3(1.1)$	$-0.14 \pm 0.037(0.07)$
P_R^u	$-1 \pm 0.15(0.19)$	$-1 \pm 0.11(1.2)$	$-1 \pm 0.15(0.24)$	$-6.0 \pm 1.4(3.3)$
P_R^d	$3 \pm 0.24(0.51)$	$-1 \pm 0.21(2.8)$	$0.5 \pm 0.09(0.48)$	$8.0 \pm 1.9(4.1)$

Table 9. Values of the couplings $P_V^e, P_L^q, P_R^u, P_R^d$ and errors as determined at NLC. 100% heavy flavour tagging efficiency and 100% longitudinal polarization of the electron beam are assumed for the first set of error-bars, while the error-bars in parentheses are for the probes without polarization.

5. Concluding remarks

In these lectures we have presented a general parametrization of an extra abelian, gauge interaction, we have commented on present limits on new gauge bosons, and we have discussed the Z' potential of LHC and NLC. Both colliders are complementary. (We did not consider t-channel limits from HERA. They are not competitive with the s-channel limits from large hadron and lepton colliders.) If a new Z' exists in the TeV range, it should be observed at LHC, where its mass and width can be measured. At a large hadron collider the strength of the new interaction and three out of four normalized charges fixing the Z' couplings to ordinary fermions can be determined for $M_{Z'} \leq 2\ TeV$. The determination of their relative signs is difficult. At NLC all the Z' couplings to quarks and leptons can be measured, including their relative signs. The average errors tend to be larger than at LHC, however. In Table 10 we give the errors for the normalized observables defined at LHC, which are implied by the NLC analysis in Table 9. These errors have to be compared with the errors expected at LHC in Table 8. For other reviews on Z' physics see Refs. [31,35].

	χ	ψ	η	LR
γ_L^l	$0.9 \pm 0.010(0.031)$	$0.5 \pm 0.04(1.5)$	$0.2 \pm 0.04(0.09)$	$0.36 \pm 0.02(0.07)$
γ_L^q	$0.1 \pm 0.017(0.045)$	$0.5 \pm 0.20(0.4)$	$0.8 \pm 0.34(1.03)$	$0.04 \pm 0.02(0.04)$
$\tilde{U}$	$1 \pm 0.30(0.38)$	$1 \pm 0.22(2.4)$	$1 \pm 0.30(0.48)$	$37 \pm 16.8(39.6)$
$\tilde{D}$	$9 \pm 1.44(3.06)$	$1 \pm 0.42(5.6)$	$0.25 \pm 0.09(0.48)$	$65 \pm 30.4(65.6)$

Table 10. Errors for the normalized observables defined at LHC implied by the NLC errors in Table 9.

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